MATH 54 - HINTS TO HOMEWORK 9

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Here are a couple of hints to Homework 9! Enjoy! :)

SECTION 7.1: DIAGONALIZATION OF SYMMETRIC MATRICES

7.1.7, 7.1.11. Remember orthogonal matrices have **orthonormal** columns!

7.1.13, 7.1.17. 7.1.22. First diagonalize the matrix as usual, and then apply Gram-Schmidt to each eigenspace!

7.1.32. Since P is orthogonal and square, $P^{-1} = P^T$. Solve for R and then calculate R^T

7.1.33, 7.1.34. Look at example 4 on page 393.

SECTION 7.2: QUADRATIC FORMS

7.2.5. Remember to evenly distribute the cross-product terms! For example, the matrix of the quadratic form $x_1^2 + 6x_1x_2 + 2x_2^2$ is:

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$$

7.2.9. Find the matrix A of the quadratic form, then diagonalize A. The matrix P of eigenvectors is the matrix you're looking for. Finally, to write the new quadratic form, use the fact that $\mathbf{x} = P\mathbf{y}$ and substitute each variable x_1, x_2 etc. with the corresponding linear combination of y_1 and y_2.

7.2.19. You don't need any linear algebra for this! Notice the following:

$$5x_1^2 + 8x_2^2 = 5x_1^2 + 5x_2^2 + 3x_2^2 = 5(x_1^2 + x_2^2) + 3x_2^2 = 5 + 3x_2^2$$

Now the largest value of x_2 is 1, hence the largest value of the quadratic form is $5{+}3 = 8$

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NOTE: In case you're stuck with the more standard differential equations questions (hom equations, undetermined coeffs, variation of parameters, etc.), make sure to look at my 'Differential equations' - Handout. That should help clarify some things!

SECTION 4.1: INTRODUCTION: THE MASS-SPRING OSCILLATOR

NOTE: If you're running out of time, **SKIP** this section! I'll let you know if a problem from that section is graded!

4.1.1. Plug in $y(t) = \sin(\omega t)$ into the equation.

4.1.3. To find the maximum value, do the standard calculus optimization trick! Calculate $y'(t)$, set it equal to 0, and solve for t.

4.1.5. The limit is 0 by the squeeze theorem

4.1.8, 4.1.9. This problem just asks you to use the method of undetermined coefficients (section 4.4)

4.1.10. Wow, what a long problem! :O You can definitely **SKIP** this if you want! It's just a matter of plugging in solutions and using the method of undetermined coefficients.

SECTION 4.2: HOMOGENEOUS LINEAR EQUATIONS: THE GENERAL SOLUTION

4.2.26. This problem looks scary, but it's not that scary! In each question, try to solve for c_1 and c_2. In (a), you'll be able to do that, in (b), there will be no solutions, and in (c) there will be infinitely many solutions!

4.2.30. Here's the trick I showed in section. Consider the Wronskian:

$$W(t) = det \begin{bmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{bmatrix}$$

You **DON'T** have to calculate this determinant explicitly. Just pick a point t_0 between 0 and 1 ($e^{-\pi}$) and calculate $W(t_0)$. If $W(t_0) \neq 0$, then your solutions are linearly independent. But if $W(t_0) = 0$, then you can't always conclude that the solutions are linearly dependent (I'll talk this some day), so usually that means pick another point that works.

SECTION 4.3: AUXILIARY EQUATIONS WITH COMPLEX ROOTS

The problems should be pretty straightforward :) Ignore $4.3.33(c)$!

SECTION 4.4: THE METHOD OF UNDETERMINED COEFFICIENTS

4.4.1. No! (in our problems, we always assumed the exponent of t to be apositive integer!)

4.4.3. Yes! $3^t = e^{\ln(3)t}$, remember that $\ln(3)$ is just a number.

4.4.5. No!

4.4.9. Guess $y_p(t) = A$

4.4.11. $y_p(t) = Ae^{2t}$

4.4.13. $y_p(t) = A\cos(3t) + B\sin(3t)$

4.4.15. $y_p(t) = (Ax + B)e^x$

4.4.17. $y_p(t) = At^2e^t$ (the double root coincides with the right-hand-side!)

SECTION 4.5: THE SUPERPOSITION PRINCIPLE

4.5.9. Yes

4.5.11. No (because of the $\frac{1}{t}$ term)

4.5.13. Actually yes! Because $\sin^2(t) = \frac{1-\cos(2t)}{2}$

SECTION 4.6: VARIATION OF PARAMETERS

Note: You *are* allowed to use your calculator of Wolfram alpha to evaluate the given integrals! This is Math 54 after all, and not Math 1B :)

The easiest way to do the problems in this section is to look at my differential equations handout!

The formula is:

Let $y_1(t)$ and $y_2(t)$ be the solutions to the homogeneous equation, and suppose $y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t)$. Let:

$$\widetilde{W}(t) = \begin{bmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{bmatrix}$$

And solve:

$$\widetilde{W}(t) \begin{bmatrix} v_1'(t) \\ v_2'(t) \end{bmatrix} = \begin{bmatrix} 0 \\ f(t) \end{bmatrix}$$

where $f(t)$ is the inhomogeneous term.

4.6.20. Don't freak out! Here we have $y_1(t) = \cos(t)$ and $y_2(t) = \sin(t)$. Just use the variation of parameters formula with f instead of the inhomogeneous term. At some point, you should get:

$$v_1'(t) = -\sin(t)f(t)$$

and

$$v_2'(t) = \cos(t)f(t)$$

Then, to get v_1 and v_2, just integrate from 0 to t:

$$v_1(t) = \int_0^t -\sin(s)f(s)ds$$

$$v_2(t) = \int_0^t \cos(s)f(s)ds$$

Finally, use the fact that $y_p(t) = v_1(t)\cos(t) + v_2(t)\sin(t)$, and use the formula $\sin(t)\cos(s) - \sin(s)\cos(t)$.